Bayesian reasoning (simplified)

$$\Pr(h \mid e) = \frac{\Pr(h)\Pr(e|h)}{\Pr(h)\Pr(e|h)+\Pr(\neg h)\Pr(e|\neg h)}$$

i. We want to calculate the probability that some hypothesis *h* is true, given some evidence *e*.

- For example: In groups who are at low risk for getting HIV, .01% of group members have HIV. The tests for HIV are very accurate. If a person has HIV, then there is a 99.99% chance that the test will come up positive, and if a person does not have HIV, there is a 99.99% chance the test will come up negative. Mary is a member of a group that has a low risk for getting HIV. She takes an HIV test and gets a positive result. Our hypothesis is that Mary has HIV. Our evidence is the positive result on the test. We want to calculate what the probability is that Mary has HIV given this result.

ii. Imagine that you were considering a large population of cases, where the population is relevantly similar to one your hypothesis is about.

- Consider 10,000 patients like Mary (I picked 10,000 because this makes the math easy given the percentages in the example).

iii. Ignoring the evidence for now, in how many of these cases will *h* be true? In how many will *h* be false. (Other terms for this: What is the *base rate* of *h*? What is the *prior probability* that *h* is true?)

- We know that 1 in 10,000 low-risk patients have HIV, so the other 9,999 will not.

iv. Look at the cases in which *h* is true. In how many of these will we see this evidence?

- The 1 patient with HIV is almost definitely going to get a positive result.

v. Look at the cases in which in *h* is not true. In how many of these will we see this evidence?

- Of the 9,999 patients without HIV, about 1 will get a positive result.

vi. The probability of *h* given evidence *e* is (iv) over (iv plus v).

- The probability is 1 / 2.

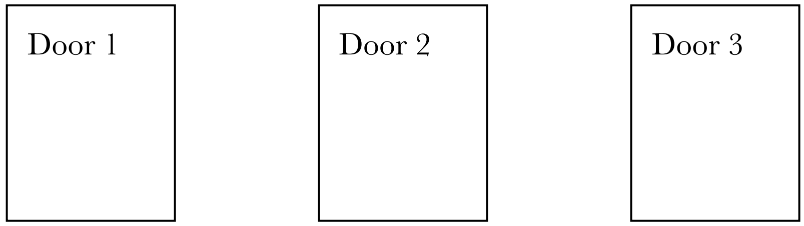
Practice questions
(from Gigerenzer, et al, Helping Doctors and Patients Make Sense of Health Statistics, 2008)

1. Assume you conduct breast cancer screening using mammography in a certain region. You know the following information about the women in this region: The probability that any randomly selected woman has breast cancer is 1%. If a woman has breast cancer, the probability that her mammograph says that she has cancer is 90%. If a woman does not have breast cancer, the probability that she nevertheless tests positive is 9%. What is the chance that a woman who tests positive on a single mammograph has breast cancer?

2. About 1 of every 800 babies born has Down's Syndrome. If a fetus with Down's Syndrome is prenatally tested, the test will report positive 82% of the time. If a fetus without Down's Syndrome is prenatally tested, the test will report positive 8% of the time. What is the chance that a fetus that tests positive for Down's Syndrome has Down's Syndrome?

3. About .3% of the population has colorectal cancer. If a person with this cancer is given a fecal occult blood test, there is a 50% chance they receive a positive result. If a person who does not have this cancer takes such a test, there is a 3% change they receive a positive result. What is the chance that someone who tests positive on a fecal occult blood test has colorectal cancer?

Let's Make a Deal

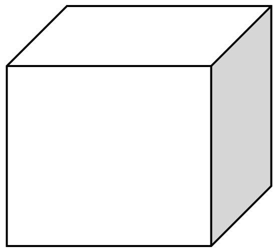


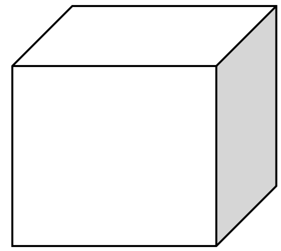
Let's Make a Deal was a television game show many years ago, hosted by Monty Hall. In one part of the show, contestants were shown three doors. Behind one of them there was a car, and nothing was behind the other two. Contestants were asked to choose the door they thought the car was behind. After they announced where they thought the car was, Monty Hall looked behind the other two doors and "eliminated" one of them, truthfully telling the contestants that there was nothing behind that door. He could only eliminate one of the doors they had not picked, and could only eliminate a door that had nothing behind it.
At this point, two doors were left – the one the contestant had picked, and one Monty Hall had not eliminated. Contestants were given a choice between staying with "their door," or switching to the other remaining door.
Once they had decided, it was revealed what was behind the door that had ultimately decided on; they got whatever it was.

For example:
There are three doors – door 1, door 2, and door 3. Unbeknownst to the contestant, the car is behind door 2. They pick door 2. Monty Hall has to eliminate door 1 or door 3; he can eliminate either, since the car is behind neither. He eliminates door 1. The contestant then has the opportunity to switch to door 3 or remain where they are.

For example:
The car is behind door 2. The contestant picks door 3. Monty Hall can only eliminate door 1 (since he can't eliminate the door the contestant picked, or the door the car is behind). He does so. The contestant now has the choice to stay with door 3 or to switch to door 1.

You are a contestant on Let's Make a Deal. You pick door 1. Monty Hall eliminates door 3. You can now stay with door 1 or switch to door 2. Which should you do?





Box A
$10,000

Box B
$?

Situation

- **Box A** has $10,000 in it.
- **Box B** has either $0 or $1,000,000 in it.

Your options

- **One box:** Take what is in **Box B only**.
- **Two box:** Take what is in **both boxes**.

The device

- The device has scanned your brain and tried to predict your choice; it knew you would be told all of this and factored that into its prediction.
- If the device predicted you would **One Box**, then it put $1,000,000 into **Box B**.
- If the device predicted you would **Two Box**, then it put nothing into **Box B**, although **Box A** still has $10,000 in it.

Other information

- This has been done very many times before and the device has made the correct prediction in the great majority of the cases.
- The device has already made its prediction and put the $0 or $1,000,000 into **Box B**.
- You know with certainty everything stated on this handout.

What choice do you make?

dominance: strategy A dominates strategy B if it is better to choose A no matter how things turn out

expected utility: roughly, the average utility that would result from a choice; the expected utility of a choice is calculated by

- determining all possible outcomes
- calculating the probability of each outcome
- calculating the utility (total goodness/badness) of each outcome
- for each outcome, multiply probability times utility; sum all of these

4. It costs \$1 to play the game. The game consists of the following: you flip a coin, and if it comes up heads, you get \$2. Otherwise, you get nothing. What is the expected value of *playing* this game?

5. It costs \$3 to play the game. The game is played as follows: You roll a regular (six sided) die once. On a six, you get \$6. On a five, you get \$5. On a four, you get \$4. On a three, you get nothing. On a two, you lose \$2. On a one, you lose \$1. What is the expected value of *playing* this game?

6. Consider two games. Both are played by flipping a fair coin; if it comes up heads, you win, and tails you lose. For both games, winning will give you double the cost of playing. Game A costs \$10 to play, and Game B costs all the money you currently have. What is the expected value of playing each game? Might it be rational for a person to be willing to play one game but not the other?